

§ 1.11

2 – 22 (even), 24, 30, 36, 40, 42

Solutions

1 – 12 Write an equation that expresses the statement

2) P is directly proportional to w .

$$P = kw$$

4) w is jointly proportional to m and n .

$$w = kmn$$

6) P varies inversely as T .

$$P = \frac{k}{T}$$

8) A is proportional to the square of t and inversely proportional to the cube of x .

$$A = \frac{kt^2}{x^3}$$

10) S is jointly proportional to the square roots of r and θ .

$$S = k\sqrt{r\theta}$$

12) A is jointly proportional to the square roots of x and y .

$$A = k\sqrt{xy}$$

13 – 22 Express the statement as an equation. Use the given information to find the constant of proportionality.

14) z varies inversely as t . If $t = 3$, then $z = 5$.

$$z = \frac{k}{t}$$

$k = 15$.

Math 1650 Homework Solutions

Jason Snyder, PhD.

16) S varies jointly as p and q . If $p = 4$ and $q = 5$, then $S = 180$.

$$S = kpq$$

$k = 9$.

18) t is jointly proportional to x and y and inversely proportional to r . If $x = 2$, $y = 3$, and $r = 12$, then $t = 25$.

$$t = \frac{kxy}{r}$$

$k = 50$.

20) H is jointly proportional to the squares of l and w . If $l = 2$ and $w = 1/3$, then $H = 36$.

$$H = k(lw)^2$$

$k = 81$.

22) M is jointly proportional to a , b , and c , and inversely proportional to d . If a and d have the same value, and if b and c are both 2, then $M = 128$.

$$M = \frac{kabc}{d}$$

$k = 32$.

24) **Law of the Pendulum**

The period of a pendulum (the time elapsed during one complete swing of the pendulum) varies directly with the square root of the length of the pendulum.

(a) Express this relationship by writing an equation.

(b) In order to double the period, how would you have to change the length l ?

$$P = k\sqrt{l}$$

$2P$ would be double the period, letting l^* be the new length, we would have

$$2P = k\sqrt{l^*}.$$

Substituting back for P yields

$$2k\sqrt{l} = k\sqrt{l^*}$$

Solving for l^* we get

$$l^* = 4l.$$

In order to double the period, we need a pendulum which is 4 times as long.

30) **Stopping Distance**

The stopping distance D of a car after the brakes have been applied varies directly as the square of the speed s . A certain car travelling at 50 mi/h can stop in 240 ft. What is the maximum speed it can be travelling if it needs to stop in 160 ft?

The equation that relates stopping distance to speed is

$$D = ks^2.$$

When $s = 50$, $D = 240$, so $k = 0.096$.

In order to stop in 160 ft, we need to solve

$$160 = 0.096s^2$$

for s . Doing so, we see that $s \approx 40.82$ mi/h.

36) **Kepler's Third Law**

Kepler's Third Law of Planetary Motion states that the square of the period T of a planet (the time it takes for the planet to make a complete revolution about the sun) is directly proportional to the cube of its average distance d from the sun.

(a) Express Kepler's Third Law as an equation.

(b) Find the constant of proportionality by using the fact that for our planet the period is about 365 days and the average distance is about 93 million miles.

(c) The planet Neptune is about 2.79×10^9 miles from the sun. Find the period of Neptune.

$$T^2 = kd^3$$

When $d = 93$ million, $T = 365$, thus $k \approx 0.166$.

To find the period of Neptune, we need to solve

$$T^2 = 0.166(2790)^3$$

for T . Do so, we get $T \approx 60043$ days.

40) **Heat of a Campfire**

The heat experienced by a hiker at a campfire is proportional to the amount of wood on the fire, and inversely proportional to the cube of his distance from the fire. If he is 20 ft from the fire, and someone doubles the amount of wood burning, how far from the fire would he have to be so that he feels the same heat as before?

The equation that relates heat intensity to amount of wood and distance is

$$H = \frac{kw}{d^3}$$

$2w$ is twice the amount of wood, and let d^* be the new distance the hiker has stand so that he feels the same heat as before.

Thus we have

$$H = \frac{2kw}{(d^*)^3}$$

Substituting back for H yields

$$\frac{kw}{20^3} = \frac{2kw}{(d^*)^3}$$

So that

$$\frac{1}{8000} = \frac{2}{(d^*)^3},$$

and therefore

$$(d^*)^3 = 16000.$$

Taking the cube root of both sides yields

$$d^* \approx 25 \text{ ft.}$$

42) **Spread of Disease**

The rate r at which a disease spreads in a population of size P is jointly proportional to the number x of infected people and the number $P - x$ who are not infected. An infection erupts in a small town with population $P = 5000$.

(a) Write an equation that expresses r as a function of x .

(b) Compare the rate of spread of this infection when 10 people are infected to the rate of spread when 1000 people are infected. Which rate is larger? By what factor?

(c) Calculate the rate of spread when the entire population is infected. Why does this answer make intuitive sense?

$$r(x) = kx(5000 - x)$$

$r(10) = k(10)(4990) = 49900k$, while $r(1000) = k(1000)(4000) = 4000000k$. The rate of spread is much faster when 1000 people are infected than when only 10 people are infected by a factor of about 80 times.

When everyone is infected $x = 5000$, so that

$$r(5000) = k(5000)(5000 - 5000) = 0.$$

This makes sense because if everyone is infected, the disease can spread to no one else.